

# Preliminary Evaluation of Radio Data Orbit Determination Capabilities for the Saturn Portion of a Jupiter-Saturn-Pluto 1977 Mission

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*Navigation accuracies attainable with radio tracking of an outer-planets spacecraft in the vicinity of Saturn have been investigated. Analysis of the results indicates that navigation accuracy problems associated with low spacecraft declinations and with batch filtering of conventional radio data when unmodeled accelerations are acting on the spacecraft may be avoided by employing range and range-rate data taken simultaneously by two widely separated stations. With the availability of two-station simultaneous data, the uncertainty in Saturn's ephemeris becomes the error source which limits the accuracy of the pre-encounter navigation.*

## I. Introduction

During the late 1970s and early 1980s, NASA intends to explore the planets Jupiter and Saturn and their environs by means of multiple-planet missions. In designing such missions it is necessary to determine the navigational capabilities commensurate with expected or possible late 1970s technology. The primary portion of any navigation study involves an accuracy analysis to establish the orbit determination capabilities associated with various spacecraft and tracking systems. The results of such a study may then be used to identify the types of tracking systems which will provide data whose inherent quality will not be a limiting error source for outer planet navigation.

A preliminary radio data accuracy analysis for the Saturn portion of a Jupiter-Saturn-Pluto 1977 (JSP77) mission has just been completed and is reported here, with

particular emphasis given to orbit determination capabilities associated with various types of radio data. A somewhat similar study for a Jupiter mission is described by Curkendall and McDanell in Ref. 1. Although the JSP77 mission has been cancelled, the orbit determination precision associated with the Saturn portion of this mission should be very representative of the precision attainable for the Saturn portion of the proposed MJS77 (*Mariner Jupiter-Saturn 1977*) mission.

To obtain preliminary estimates of radio navigation capabilities, covariance analyses were performed for orbit determination solutions based upon the following types of radio data:

- (1) Conventional range and range-rate (doppler).
- (2) Multi-station range-rate and simultaneous range.

- (3) Explicitly differenced range and range-rate.

The main concern of the accuracy analysis was establishing the sensitivities of the solutions to the following quantities:

- (1) Constant station location errors.
- (2) Constant and stochastic spacecraft accelerations.
- (3) Data coverage.
- (4) Length of data arc.
- (5) *A priori* knowledge of the planetary ephemeris.

The orbit determination capabilities are expressed in the usual B-plane coordinates shown in Fig. 1 and described in Ref. 2.

If it is desired that the pre-encounter radio-only solution accuracies be less than 1000 km ( $1\sigma$ ), the results of this preliminary accuracy analysis lead to the following conclusions:

- (1) If the spacecraft has a low declination, simultaneous or nearly simultaneous high precision (3-m) range data will be required.
- (2) If the spacecraft is subject to unmodeled accelerations larger than  $10^{-14}$  km/s<sup>2</sup> during much of the data arc, simultaneous range and range-rate data will be required.

With the availability of simultaneous data to alleviate the degradation of orbit determination precision due to low declinations and unmodeled stochastic accelerations acting on the spacecraft, the planetary ephemeris uncertainty becomes the dominant error source affecting the accuracy of the pre-encounter estimates of the planet-relative spacecraft state.

It has also been concluded that the mass and ephemeris of Saturn should be estimated if optimal navigation accuracies are to be achieved. Thus, improved knowledge of these parameters will be a by-product of the spacecraft orbit determination procedure. The amount of improvement resulting from the use of different types of radio data is discussed in some detail in another article.<sup>1</sup>

<sup>1</sup>"Determining the Mass and Ephemeris of Saturn by Radio Tracking of a Jupiter-Saturn-Pluto 1977 Spacecraft" by V. J. Ondrasik, et al., in this issue.

<sup>2</sup>P. A. Penzo, JPL Systems Analysis Section, private communication.

## II. Trajectory and Data

The Earth mean equatorial cartesian and B-plane coordinates for the particular JSP77 trajectory that was investigated are given in Tables 1 and 2 and were supplied by P. Penzo.<sup>2</sup> The temporal behavior of the Earth to spacecraft spherical coordinates is shown in Fig. 2 over the tracking period of encounter ( $E$ ) - 120 days to  $E + 30$  days. The most interesting feature of this figure involves the spacecraft declination which at encounter is almost zero and never rises above 6 deg.

In the integration of the trajectory and variational equations, and the subsequent calculation of data and the associated partial derivatives, the following assumptions were made:

- (1) Only the Sun and Saturn accelerate the spacecraft.
- (2) Saturn and Earth move in ellipses about the Sun.
- (3) The velocity of light equals infinity.

Previous studies have shown that these assumptions will not significantly affect the accuracy analysis results.

In anticipation that data taken simultaneously or nearly simultaneously from two stations will be useful, the simulation tracking pattern shown in Fig. 3 was based upon a "cycle" of data. As shown in this figure, a cycle consists of range and range-rate measurements taken from Deep Space Stations (DSSs 14 (California), 42 (Australia), 61 (Spain), and 14 again. Each pass includes horizon-to-horizon range-rate data taken on the hour, and usually two range points. During the overlap period between stations, the range-rate points will be taken at exactly the same times, and the range point from one station will occur within a few minutes of a range point taken from another station. In practice, the simultaneous range-rate and range information will be obtained by using two-way and three-way doppler data, respectively. If the round-trip light time is less than the overlap time, the information supplied by simultaneous range points may be obtained from nearly simultaneous ( $\sim 15$ -min separation) two-way range points. Operationally, precision two-way range points are easier to obtain than three-way range points because problems associated with the synchronization of station clocks are avoided (Ref. 3, pp. 118-119).

## III. Data Weights

Although only one range-rate point is taken per hour during a tracking pass, each point is weighted so that it is representative of range-rate points taken once a minute

with a data weight of 1 mm/s. This weight implies that for the outer planet missions both the two- and three-way doppler data will be of the same quality as the best present two-way data.

As shown in Ref. 3, pp. 120–122, the reason for taking precise simultaneous range data is to avoid problems associated with spacecraft at low declinations. Thus, when range data from more than one station are included in the orbit determination solution, these data will be given a weight of 3 m. When the solution contains range data from only one station, they will be given a range weight of 300 m to diminish the errors produced by unmodeled spacecraft accelerations as discussed in Ref. 4. Data weights are data accuracy specifications provided to the orbit determination solution filter, and should be distinguished from expected measurement precision. For example, range data, of a given quality, may be assigned different weights, depending on how well the spacecraft and tracking system can be modeled.

In later sections it will become apparent that the main reason for taking simultaneous range and range-rate data is to diminish orbit determination errors produced by unmodeled spacecraft accelerations. As shown in Ref. 3, pp. 123–124, new data types which are formed by explicitly differencing the simultaneous (or near-simultaneous) range and range-rate data are insensitive to such accelerations. In this study the differenced range and range-rate data are assigned weights of 3 m and 1 mm/s, respectively. Since the differenced data contain very little information about the geocentric range and range-rate, the orbit determination solution must also include some conventional range and range-rate data. However, to insure that these conventional data are used only in the determination of the geocentric range and range-rate components and do not introduce spacecraft acceleration information, they are assigned very loose weights. In this study the conventional range and range-rate data were given weights of 10 km and 100 mm/s, respectively, in the differenced data orbit determination solutions.

#### IV. Error Sources

In addition to the data noise, two other sources of error are assumed to be present, namely, constant station location errors and unmodeled spacecraft accelerations. It is assumed that the effects of transmission media, polar motion, and other such error sources as well as the actual crust-fixed station location biases can be represented to some degree by considering the effects of constant station

location errors. As shown in Table 3, the assumed errors in the station's cylindrical coordinates, i.e., distance off the spin axis ( $r_s$ ), longitude ( $\lambda$ ), and distance from the equator ( $z_s$ ), are 1 m, 2 m, and 5 m, respectively. The errors in the position and velocity of the spacecraft which are produced by these constant station location errors were evaluated by using the Double-Precision Orbit Determination Program (DPODP) consider option described in Ref. 5, pp. 109–116. The numbers expressing navigational capabilities given in the following sections could be lowered if the station locations were estimated instead of being considered to be in error by the above amounts. However, experience has shown that only under limited circumstances is it possible to solve for station locations without degrading the solution of the position and velocity of the spacecraft.

The unmodeled spacecraft accelerations are represented with adequate generality as a three-axis, exponentially time-correlated stationary process. As shown in Table 3, the standard deviation of these accelerations is assumed to be  $10^{-12}$  km/s<sup>2</sup> with a correlation time  $\tau$  of either 6 days or infinity. The case of  $\tau = \infty$ , of course, corresponds to a constant acceleration. The analysis technique used to determine the sensitivity of the conventional least-squares batch filter solution for the spacecraft state and planetary mass and ephemeris to the unmodeled spacecraft accelerations is that developed by Curkendall in Ref. 6. If any part of the accelerations were constant or nearly so, it might be possible to estimate these accelerations and thereby diminish the errors they would produce in the solution for the spacecraft's position and velocity. For this reason the list of estimated parameters will often be extended to include three constant spacecraft accelerations.

#### V. Solution Filter, Estimated Parameters, and A Priori

With only one exception, the orbit determination solutions were obtained with a conventional least-squares batch filter (Ref. 5) estimating the spacecraft state, planetary mass, planetary ephemeris, and constant spacecraft accelerations. The heliocentric *a priori* knowledge of the spacecraft position and velocity was always taken to be  $10^7$  km and 1 km/s, respectively. The *a priori* knowledge of the constant accelerations was taken as  $10^{-12}$  km/s<sup>2</sup>.

The planetary ephemeris was solved for in terms of Brouwer and Clemence Set III elements described in Ref. 5, pp. 26–29, and Ref. 7. The nominal *a priori* knowledge of the mass and Brouwer and Clemence Set III elements

are given in Table 4 and were recommended by Lieske.<sup>3</sup> In order to obtain some idea of how the ephemeris *a priori* influences the orbit determination process, covariance analyses were also performed for solutions with an improved *a priori* knowledge of the planetary ephemeris. The improved *a priori* ephemeris is also shown in Table 4 and is down from the nominal by a factor of three. This nominal ephemeris *a priori* will probably be available if the current level of ephemeris work continues. However, improvement of the nominal *a priori* by a factor of three will require a much larger effort. Figure 4 shows the contribution of the nominal *a priori* ephemeris to the Saturn-centered Earth equatorial B-plane errors. It should be noted that the nominal *a priori* ephemeris uncertainty gives a semi-major axis of the error ellipse of approximately 1200 km.

## VI. Error Analyses Sets

In order to obtain some idea of the orbit determination capabilities of the Saturn portion of the JSP77 mission using radio data only, consider covariances were computed for many cases involving different data types, data arc lengths, error sources, *a priori* uncertainties, data coverage, and estimated parameters. These cases are grouped into sets and described in Table 5. Each set is chosen to study one particular feature of the orbit determination procedure.

The orbit determination capabilities are expressed in terms of the semi-major axis (SMAA) and semi-minor axis (SMIA) of the B-plane  $1\text{-}\sigma$  error ellipse, and the standard deviations of the magnitude of the  $\mathbf{B}$ -vector ( $\sigma_B$ ) and the linearized time of flight ( $\sigma_t$ ). Generally the orbit determination capabilities are given for the data arcs terminating at  $E - 11$  days,  $E - 5$  days,  $E - 2$  days,  $E + 4$  days, and  $E + 28$  days.

## VII. Errors in State Only Solutions Produced by Various Error Sources

To obtain some idea of the size of the spacecraft state errors produced individually by errors in Saturn's mass and ephemeris, station locations, and nongravitational accelerations, a sensitivity matrix was computed for case 1 of Table 5. (See Ref. 8 for a discussion of the sensitivity matrix.) Figure 5 shows the errors in the estimates of  $\mathbf{B} \cdot \mathbf{R}$ ,  $\mathbf{B} \cdot \mathbf{T}$  and linearized time of flight resulting from the

indicated errors associated with various parameters for the solutions made with three-station data arcs starting at  $E - 120$  days and terminating at  $E - 5$  days and  $E - 2$  days. These errors in the B-plane quantities scale directly with the magnitude of the errors from each source.

The sensitivity matrix will change for each different solution. However, the following gross conclusions may be drawn from these results:

- (1) It will be necessary to solve for planetary mass and ephemeris.
- (2) Constant spacecraft accelerations must be estimated if they are expected to be present at a level higher than  $10^{-14}$  km/s<sup>2</sup>.
- (3) Station location errors are not a major concern (primarily since the solution uses simultaneous range).

## VIII. Long Arc Solutions Using Single Station, Multi-Station, and Differenced Data

### A. Conventional Consider Analysis

To compare the orbit determination capabilities of long data arc (i.e., data starts at  $E - 120$  days) solutions using three-station data, single-station data, and explicitly differenced data, error analyses were performed on cases 2, 3, and 4 of Table 5. The one-sigma conventional consider standard deviations, which account for the formal statistics resulting both from data noise only and from constant station location errors, are shown for the pertinent B-plane parameters in Fig. 6 for several different data arc lengths.

The most notable feature of these results is that, irrespective of the data types, for data arcs extending up to  $E - 2$  days, the SMAA is not reduced below its value given by the *a priori* ephemeris uncertainty shown in Fig. 4. However, the solutions based upon multi- or single-station tracking do contain some planet-centered information as is evidenced by the improvement in SMIA and  $\sigma_B$  for data arcs terminating at  $E - 5$  days or beyond. The differenced data solutions do not show a corresponding reduction in  $\sigma_B$  and SMIA because, as explained in Ref. 3, pp. 123-124, the explicit differencing procedure deletes almost all of the planet-centered acceleration information. When a few days of post-encounter data are included in the solution, the SMAA is reduced below 10 km for solutions based upon all three data types. The post-encounter differenced data solutions take a longer time to improve than do the solutions which employ the conventional data

<sup>3</sup>J. H. Lieske, JPL Tracking and Orbit Determination Section, private communication.

types because the differenced data obtain planet-relative information only indirectly through changes in the spacecraft-planet-Earth geometry.

Figure 6 also shows that, for data arcs ending at  $E - 11$  days or before, the SMAA,  $\sigma_B$ , and SMIA are generally a factor of two larger for the solutions based upon single-station data than for solutions based upon either multi-station or differenced data. This degradation of the solution is a manifestation of the zero-declination problem and is eliminated by the use of simultaneous or nearly simultaneous range data.

Another notable feature is that the time of flight is determined much better from conventional data. Presumably this occurs because much of the time of flight information is supplied by the planet accelerating the spacecraft. A curious result exhibited in Fig. 6 is that the solution based upon multi-station data may degrade from earlier solutions in certain directions if the data arc terminates a few days before encounter. This degradation appears to be a manifestation of a phenomenon that has been observed before; namely, as the information supplied by the planetary-induced accelerations starts to become important, it competes with the information supplied by the spinning Earth, and under certain circumstances the covariance is degraded. This effect has sporadically been the subject of casual investigations, but presently is not well understood.

## B. Effects of Unmodeled Accelerations

So far in this section the only factors that have been assumed to degrade the orbit determination solution have been data noise and constant station location errors. An additional error source which is very important is stochastic unmodeled spacecraft accelerations. Such error sources can be particularly damaging because the batch filter is incapable of modeling such a process. The effect of the unmodeled accelerations on the solutions based upon single-station, three-station, and differenced data are shown in Fig. 7 for several data arc lengths. This figure contains only the effects due to unmodeled accelerations with a correlation time of 6 days as described in Table 4.

It is immediately apparent that for solutions based upon three-station data with a 3-m range weight the effects of unmodeled accelerations can be catastrophic. For example, for a multi-station data arc terminating at  $E - 4$  days, the stochastic accelerations produce a  $30000\text{-km} \times 7000\text{-km} \times 200\text{-second}$  B-plane error ellipsoid. The effects of unmodeled accelerations on the single-station solution are typically an order of magnitude less damaging than

the effects on the multi-station solutions. For a solution based upon a single-station data arc terminating at  $E - 4$  days, these accelerations produce a  $4000\text{-km} \times 2000\text{-km} \times 40\text{-second}$  B-plane error ellipsoid.

If the solutions are based upon explicitly differenced data, the effects of unmodeled spacecraft accelerations are practically negligible. For example, for a solution based upon a differenced data arc terminating at  $E - 4$  days, stochastic accelerations produce only a  $100\text{-km} \times 30\text{-km} \times 5\text{-second}$  B-plane error ellipsoid. These errors are two or three orders of magnitude smaller than the ones resulting from conventional data because, as mentioned earlier, the differencing process deletes almost all of the information which is sensitive to the spacecraft accelerations.

With a few days of post-encounter data in the solution, the stochastic acceleration-induced error ellipsoid is reduced to approximately  $20\text{ km} \times 20\text{ km} \times 1\text{ second}$  for all three data sets.

## IX. Short Arc Solutions

### A. Conventional Consider Analysis

The previous section was concerned with orbit determination solutions based upon a "long" arc of data, i.e., the data started at  $E - 120$  days. In an attempt to determine if a data arc of this length is necessary, covariance analyses were performed for cases identical to cases 2, 3, and 4 except that data acquisition was initiated at  $E - 34$  days, i.e., a "short" arc of data. These short arc cases are described in Table 5. The B-plane errors produced by data noise and station location errors for solutions based upon short arcs of three-station, single-station, and differenced data are shown in Fig. 8.

The most notable feature of this figure is that, for data arcs terminating at  $E - 11$  days, the SMAA associated with single-station tracking is approximately five times larger than the SMAA associated with either multi-station or differenced data. This degradation of the single-station data solution is a manifestation of the zero-declination problem, which will be present if simultaneous range data are not used.

Another feature of particular interest is that, for solutions based upon data arcs extending to within five days of encounter, the short arc B-plane error ellipsoids are nearly the same as those associated with the analogous long arc cases.

## B. Effects of Unmodeled Accelerations

The parameters describing the B-plane error ellipsoid produced by the unmodeled stochastic accelerations described in Table 3 for three-station, single-station, and differenced data solutions are shown in Fig. 9. As was the case for long arc solutions, the effects of unmodeled accelerations on the single-station and three-station solutions can be disastrous, producing errors of thousands of kilometers. Once more, however, if differenced data are used, the effects of the spacecraft accelerations on the orbit determination solution will be negligible.

## X. Ephemeris A Priori Dependence

In previous sections it has been shown that solutions based upon pre-encounter data yield a SMAA for the B-plane error ellipse which is due almost entirely to uncertainties in the planetary ephemeris. To obtain some idea of how much the planet-centered spacecraft state improves with better *a priori* knowledge of the planetary ephemeris, the pertinent error analyses were performed. These cases are described in Table 5.

Figure 10 contains the B-plane error ellipsoid parameters for three-station and differenced data solutions made with the nominal and with  $\frac{1}{3}$  nominal or improved ephemeris *a priori* described in Table 3. It is easily seen that improving the *a priori* knowledge of the planetary ephemeris by a factor of three generally produces a reduction in the pre-encounter SMAA by approximately the same factor.

## XI. Data Coverage Dependence

To grossly determine the amount of data coverage which will be required, error analyses were performed for a solution which was identical to the long arc three-station data solution (case 2 of Table 5), except half of the data

was deleted. The particular portion of the data which was deleted is indicated by the deletion code in Fig. 3. A comparison of the B-plane parameters associated with the full data coverage and "one-half" data coverage shows differences which are always less than 10%. Thus, it is probably sufficient to use only one-half the data coverage shown.

## XII. Summary and Discussion

A study of the results which have been presented in the preceding sections leads to the following tentative conclusions regarding the radio data orbit determination procedure for outer planet missions:

- (1) For solutions made without any data taken after encounter minus a few days, the spacecraft state uncertainty is reduced only slightly from the uncertainty associated with the planetary *a priori* ephemeris.
- (2) If the spacecraft is at a low declination, short arc solutions are substantially improved with the use of simultaneous or near-simultaneous precision range data.
- (3) If the spacecraft is undergoing unmodeled accelerations larger than  $10^{-14}\text{km/s}^2$ , it will probably be necessary to use explicitly differenced simultaneous data.

Thus, with the availability of simultaneous data, the planetary ephemeris uncertainty becomes the dominant error source and the errors associated with zero-declination geometry and/or unmodeled spacecraft accelerations become almost negligible.

As is the case with all accuracy analyses, the results shown in the preceding sections are based upon particular error models and thus are representative of the actual accuracies obtainable only to the degree to which the error model is representative of the true error sources.

## References

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**Table 1. Saturn, Earth, and spacecraft coordinates at encounter**

Coordinate	Object			
	Earth (heliocentric)	Saturn (heliocentric)	Spacecraft (heliocentric)	Spacecraft (planet centered)
$X$	$0.93054 \times 10^8$	$-0.14201 \times 10^{10}$	$-0.14203 \times 10^{10}$	$-0.13472 \times 10^6$
$Y$	$0.10558 \times 10^9$	$-0.66544 \times 10^8$	$-0.6624 \times 10^8$	$0.29559 \times 10^6$
$Z$	$0.45810 \times 10^8$	$0.33937 \times 10^8$	$0.33416 \times 10^8$	$-0.52131 \times 10^6$
$\dot{X}$	$-0.23651 \times 10^2$	$-0.20706 \times 10^0$	$-0.17789 \times 10^2$	$-0.17583 \times 10^2$
$\dot{Y}$	$0.17084 \times 10^2$	$-0.89422 \times 10^1$	$-0.15492 \times 10^2$	$-0.65502 \times 10^1$
$\dot{Z}$	$0.74133 \times 10^1$	$-0.36914 \times 10^1$	$-0.15126 \times 10$	$0.21789 \times 10^1$

**Table 2. Equatorial B-plane coordinates**

Coordinate	Value
$B$	$0.7686 \times 10^6$
$\underline{B} \cdot \underline{R}$	$0.6479 \times 10^6$
$\underline{B} \cdot \underline{T}$	$0.4135 \times 10^6$
$C_3$	$0.2352 \times 10^2$
$\phi$	57.45 deg

**Table 3. Error sources**

Error source	Time behavior	Magnitude
Stations:		
(1) Distance off spin axis	Constant	1 m
(2) Longitude	Constant	2 m
(3) Distance off equator	Constant	5 m
Spacecraft accelerations:		
(1) Constant	Constant	$10^{-12}$ km/s <sup>2</sup>
(2) Stochastic	Piece-wise constant for 2 days with correlation time of 6 days	Standard deviation of distribution = $10^{-12}$ km/s <sup>2</sup>



**Table 4. Nominal and improved planetary mass and ephemeris a priori**

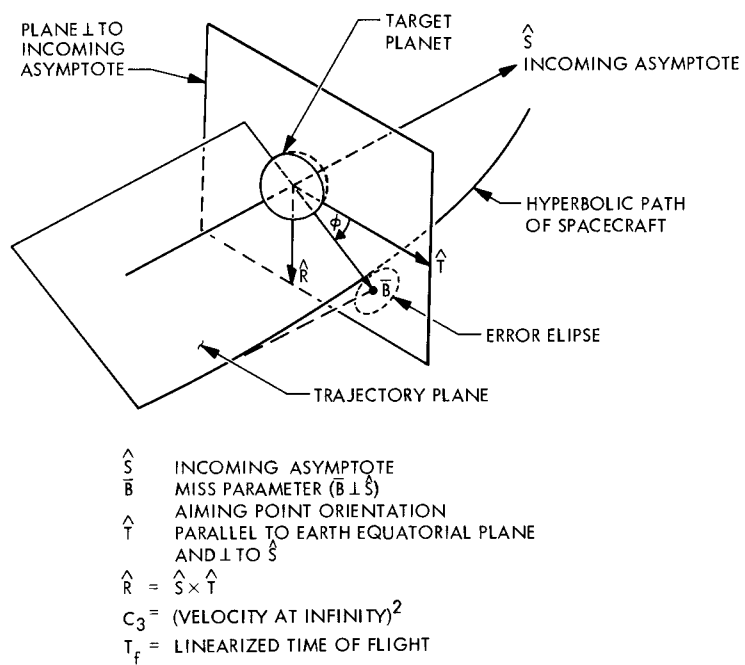
Planet	$GM$ <i>a priori</i> , km <sup>3</sup> /s <sup>2</sup>	$10^2 \frac{\sigma_{GM} (a priori)}{GM}$ , %	Ephemeris parameter	Ephemeris <i>a priori</i> , rad		$\sigma$ (ephemeris), $1.5 \times 10^9$ km	
				Nominal	Improved	Nominal	Improved
Saturn	$0.3 \times 10^5$	0.08	DA	$0.6 \times 10^{-7}$	$0.2 \times 10^{-7}$	$0.9 \times 10^2$	$0.3 \times 10^2$
			DE	$0.3 \times 10^{-6}$	$0.1 \times 10^{-7}$	$0.45 \times 10^3$	$0.15 \times 10^3$
			DMW	$0.6 \times 10^{-6}$	$0.2 \times 10^{-6}$	$0.9 \times 10^3$	$0.3 \times 10^3$
			DP	$0.3 \times 10^{-6}$	$0.1 \times 10^{-6}$	$0.45 \times 10^3$	$0.15 \times 10^3$
			DQ	$0.3 \times 10^{-6}$	$0.1 \times 10^{-6}$	$0.45 \times 10^3$	$0.15 \times 10^3$
			EDW	$0.3 \times 10^{-6}$	$0.1 \times 15^{-6}$	$0.45 \times 10^3$	$0.15 \times 10^3$

**Table 5. Case numbers and solution sets**

Case	Set	Data <sup>a</sup>	Stations	Ephemeris <i>a priori</i> <sup>b</sup>
1	Estimate state only	$\rho$ and $\dot{\rho}$	14, 42, 61	—
2	Long arc solutions	$\rho$ and $\dot{\rho}$	14, 42, 61	Nominal
3		$\rho$ and $\dot{\rho}$	14 only	Nominal
4		Diff	14, 42, 62	Nominal
5	Short arc solutions	$\rho$ and $\dot{\rho}$	14, 42, 61	Nominal
6		$\rho$ and $\dot{\rho}$	14 only	Nominal
7		Diff	14, 42, 61	Nominal
8	Ephemeris-dependence solutions	$\rho$ and $\dot{\rho}$	14, 42, 61	Improved
9		Diff	14, 42, 61	Improved

<sup>a</sup> $\rho$  = range;  $\dot{\rho}$  = range-rate; diff = explicitly differenced data.

<sup>b</sup>See Table 4.



**Fig. 1. B-plane coordinate system**

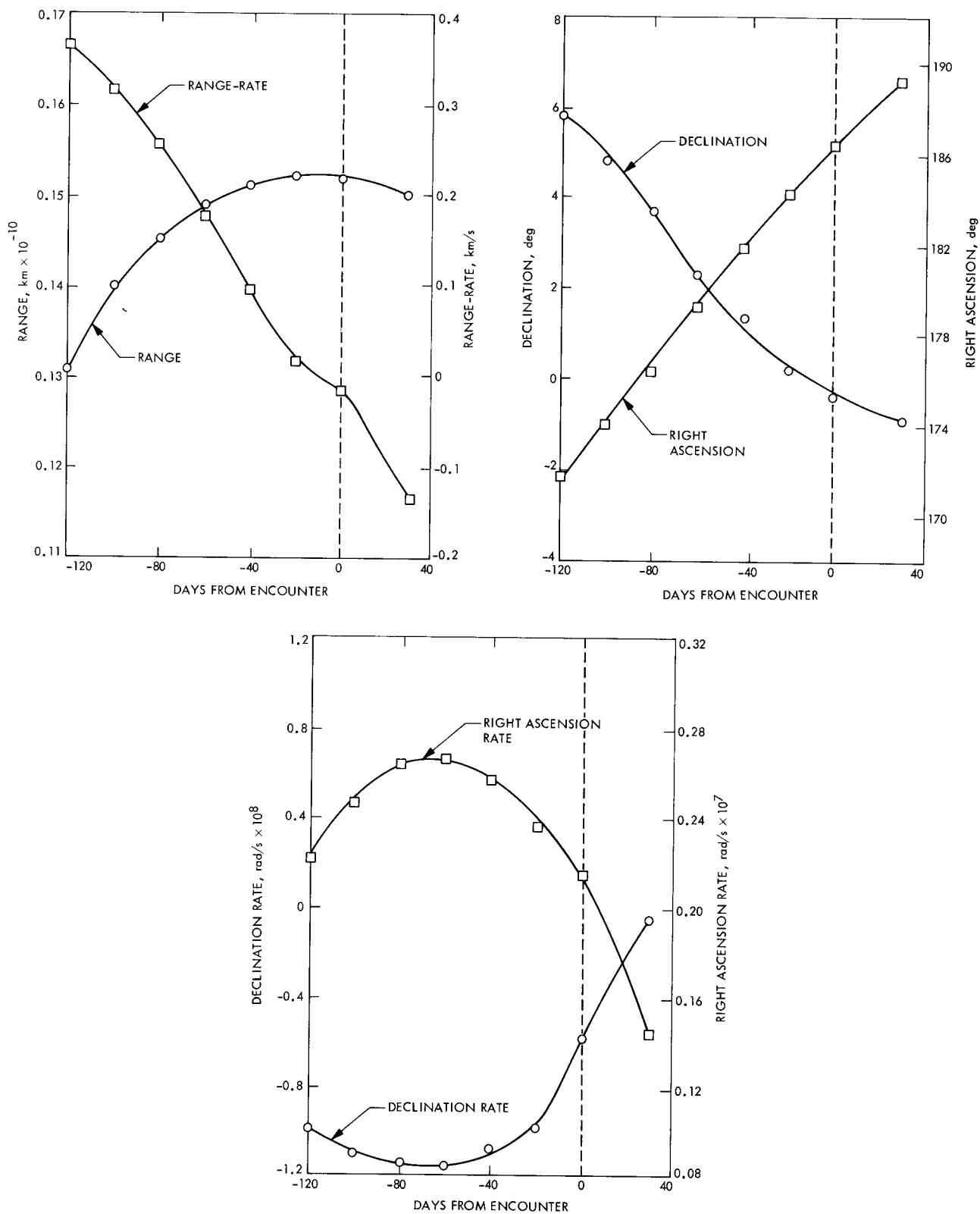


Fig. 2. Earth-spacecraft spherical coordinates

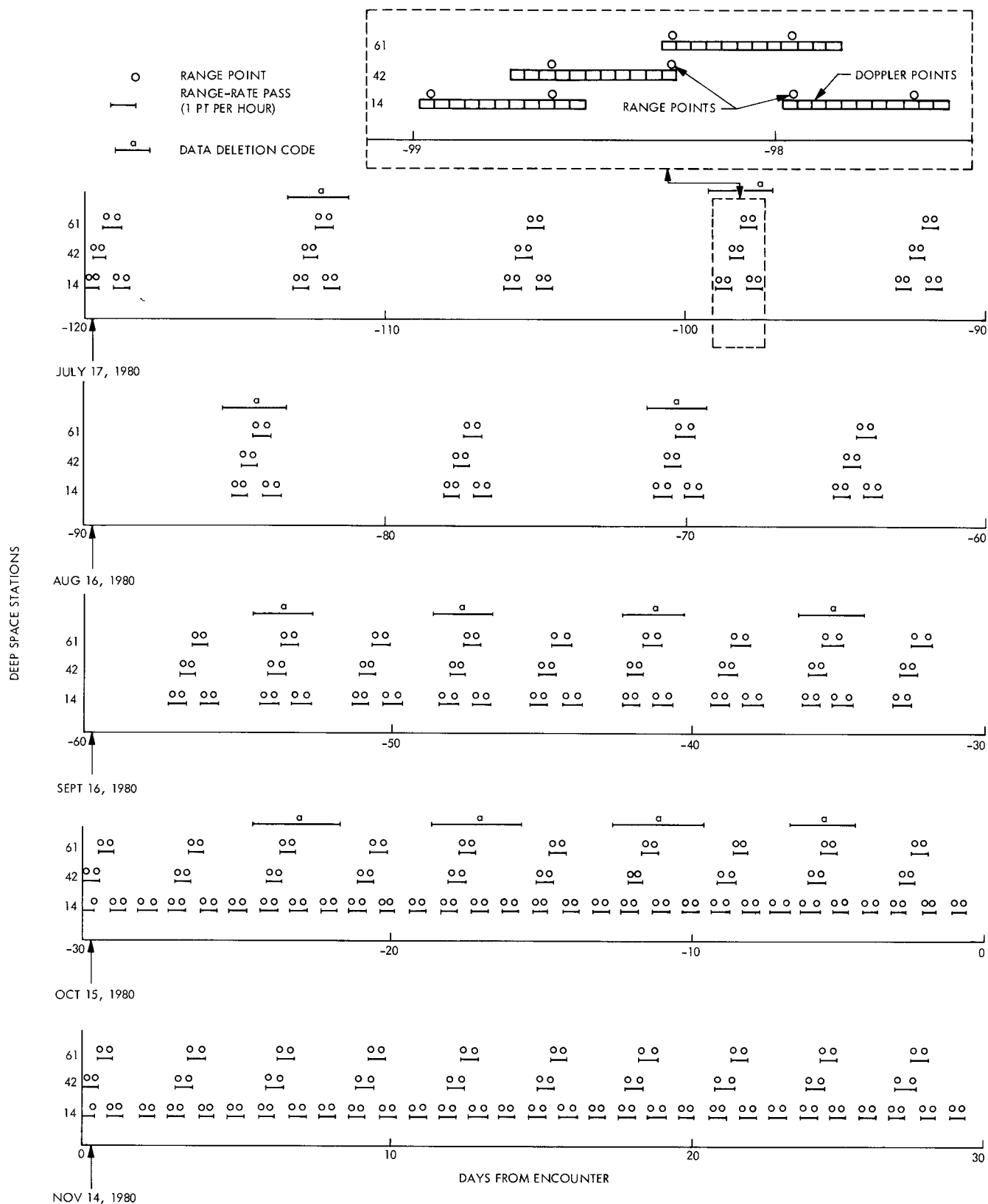
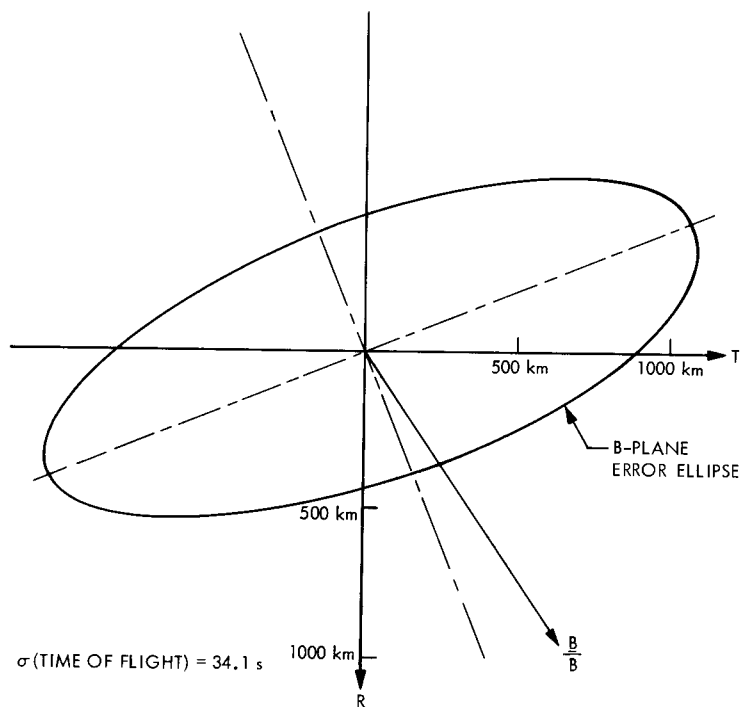


Fig. 3. DSSs 14, 42, and 61 tracking patterns



**Fig. 4. A priori ephemeris uncertainties mapped into the linearized time of flight and the equatorial B-plane ellipse**

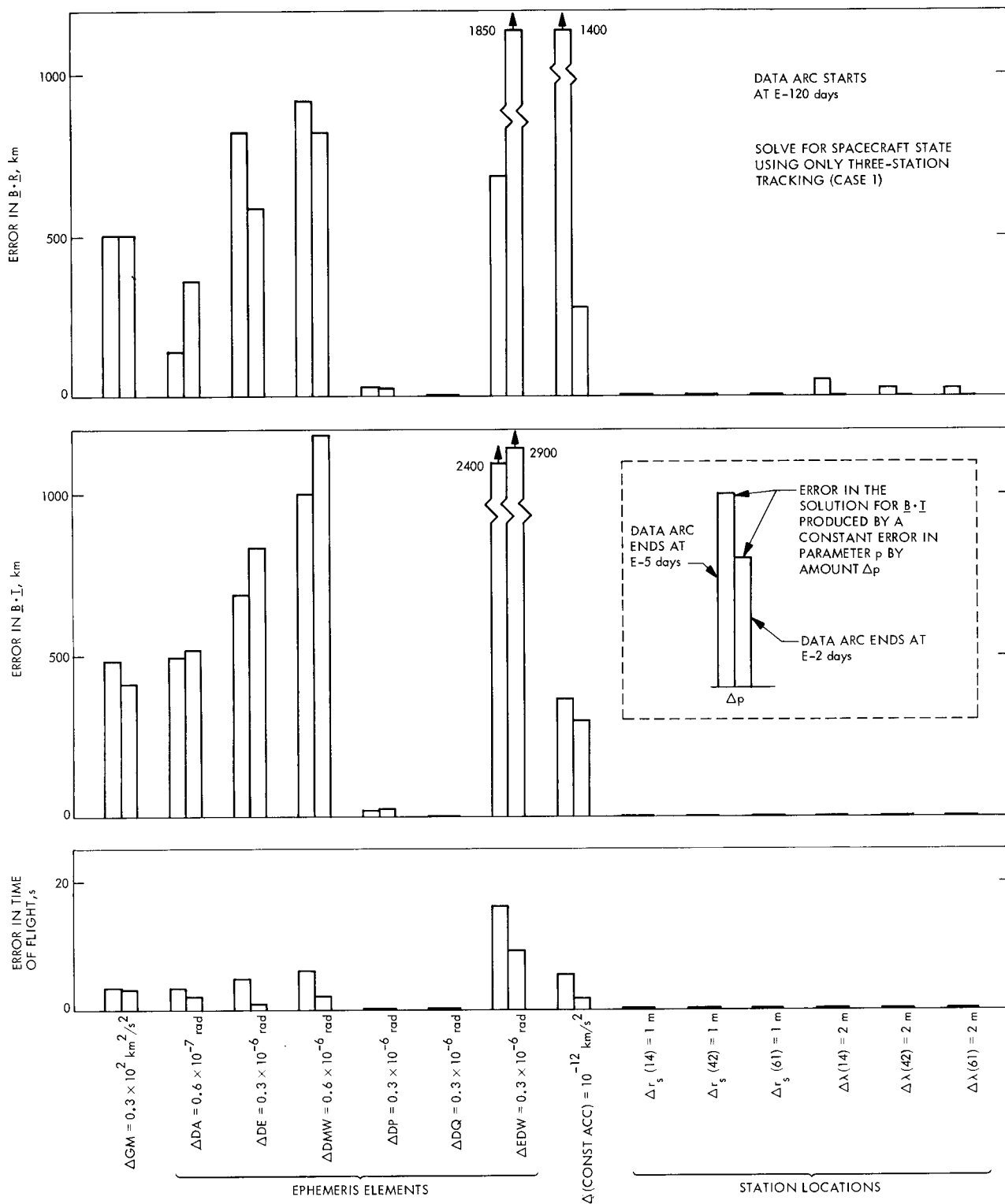
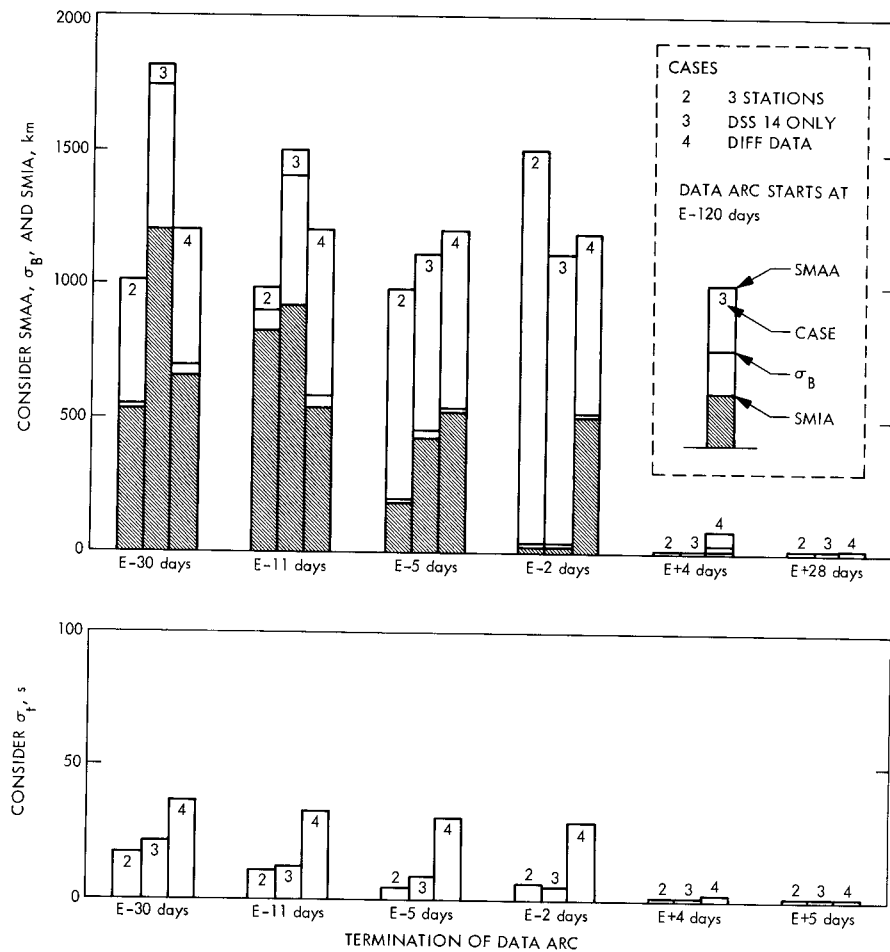


Fig. 5. Errors in B-plane parameters produced by constant error sources for a solution based upon a long arc of three-station data



**Fig. 6. Effects of data noise and station location errors on long arc solutions**

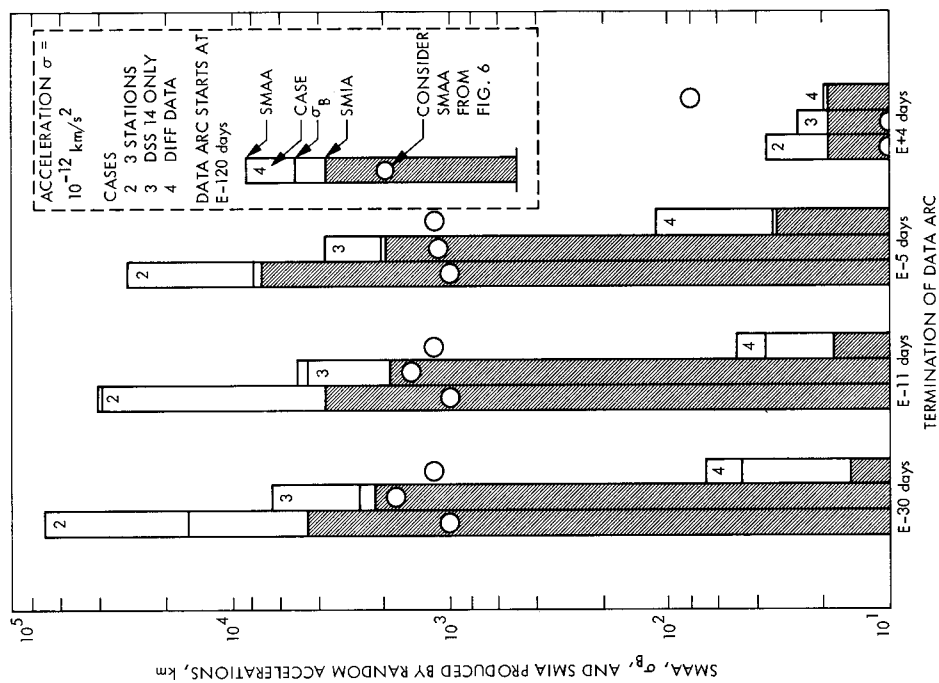


Fig. 7. Effects of unmodeled random accelerations only on long arc solutions using three-station, DSS 14 only, and differenced data

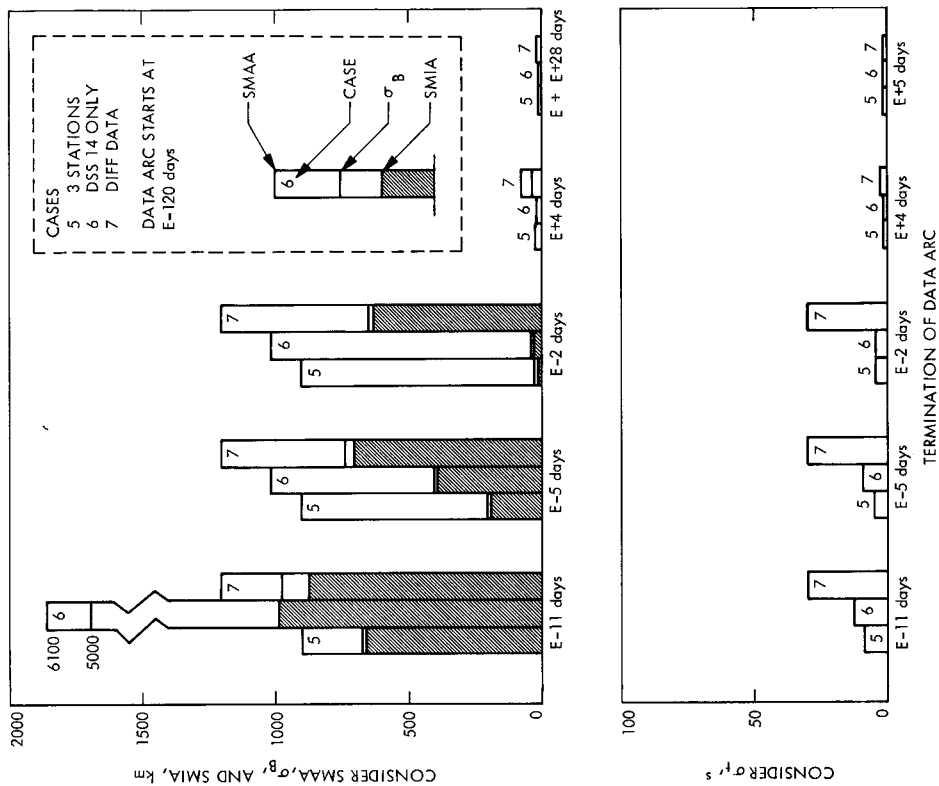


Fig. 8. Effects of data noise and station location errors on short arc solutions



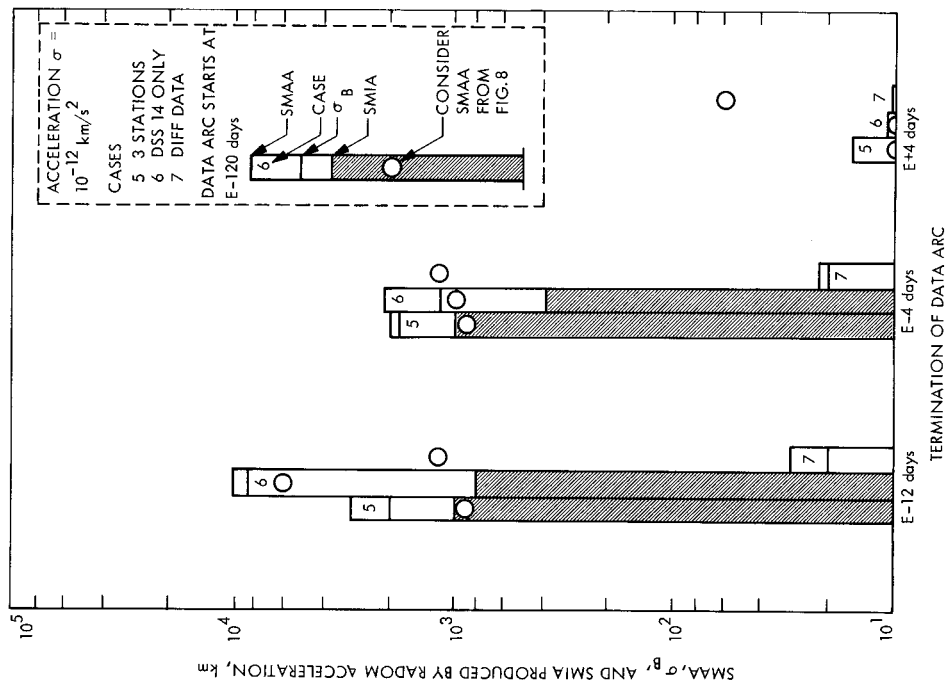


Fig. 9. Effects of unmodeled random accelerations only on short arc solutions using three-station, DSS 14 only, and differenced data

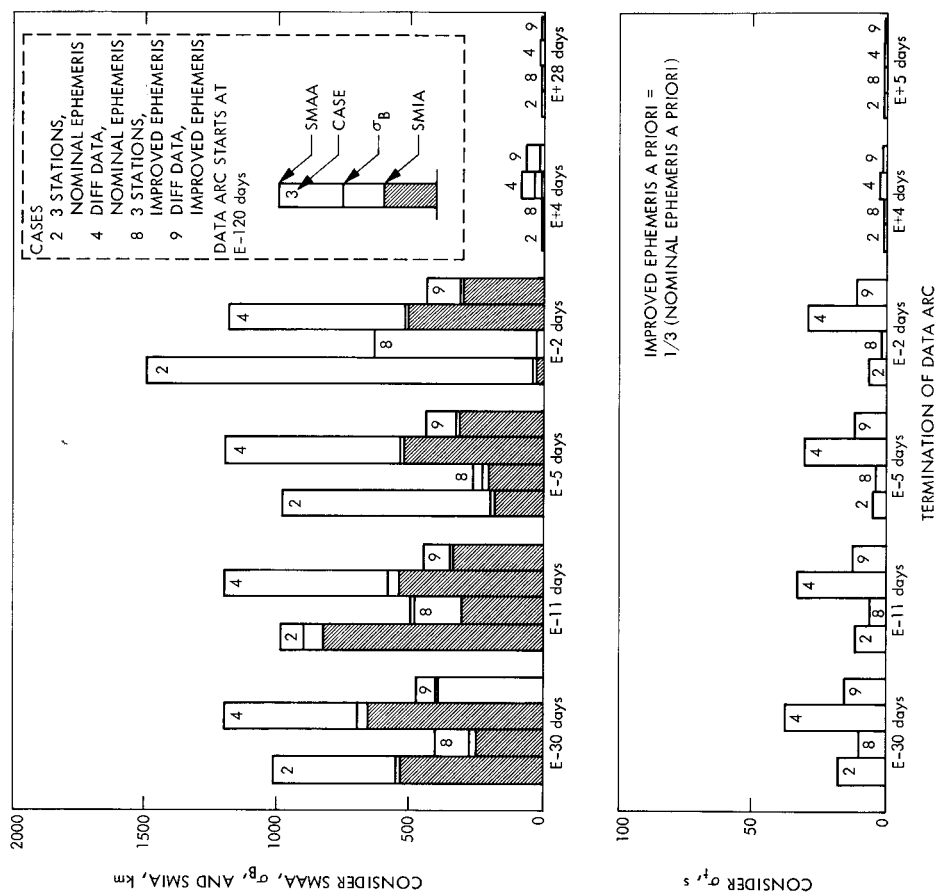


Fig. 10. Effect of planetary ephemeris a priori on long arc solutions